4.2 Conduction Heat Transfer

For one dimensional (1D) for homogeneous, isotropic solids:

Coordinate System	Governing Equation	Thermal condition	Examples
Cartesian	$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$	Transient	Slab, Plane wall
Cylindrical	$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$	Transient	Solid/hollow pipe, tube, wire, etc.
Spherical	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$	Transient	Solid & hollow sphere

$\dot{e}_{gen}$ → Volumetric heat generation, W/m^3
k → Thermal conductivity, W/m.K
α → Thermal diffusivity = k/ρc, m^2/s

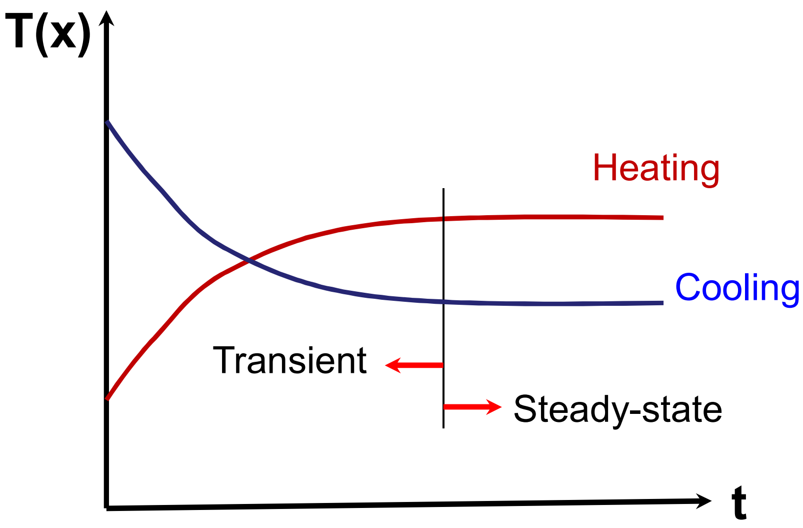
4.2 Conduction Heat Transfer

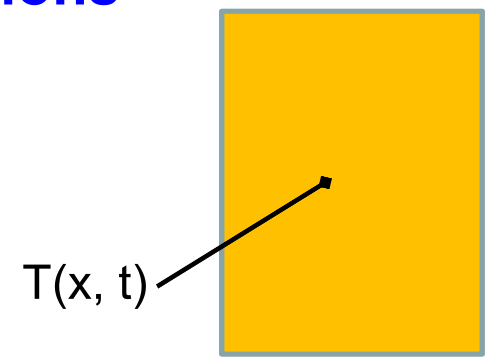
For one dimensional (1D) homogeneous, isotropic solids **without heat generation**:

Coordinate System	Governing Equation (Transient)	Governing Equation (Steady State)
Cartesian	$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$	$\frac{d^2 T}{dx^2} = 0$
Cylindrical	$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$	$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$
Spherical	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$	$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$

k → Thermal conductivity, W/m.K
α → Thermal diffusivity = k/ρc, m^2/s

Transient versus Steady-State conditions





ME265: Thermal Engineering & Heat Transfer

Chapters
1. Energy Scenario
2. Thermodynamics
3. Mechanical Devices & Systems
4. Heat Transfer

4.1 Introduction	
4.2 Conduction	4.2.1 Conduction Equations 4.2.2 Boundary & Initial conditions 4.2.3 Steady Heat Conduction 4.2.4 Transient Heat Conduction
4.3 Convection	
4.4 Radiation	
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4.2.2 Initial and Boundary Conditions

Conduction Equation/
Heat Diffusion Equation/
Governing Equation

Solution →

Temperature distribution within a solid

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

We need → T(x,t)

Two boundary conditions
One initial condition

$$\frac{d^2 T}{dx^2}$$

We need → T(x)

Two boundary conditions only

4.2.2 Initial and Boundary Conditions

2.2. Boundary and initial conditions

Initial condition:

Temperature in the medium at t=0: **T(x,0), T(r,0)**

Boundary conditions:

- Thermal conditions at the boundaries of the medium
- BCs are of various types

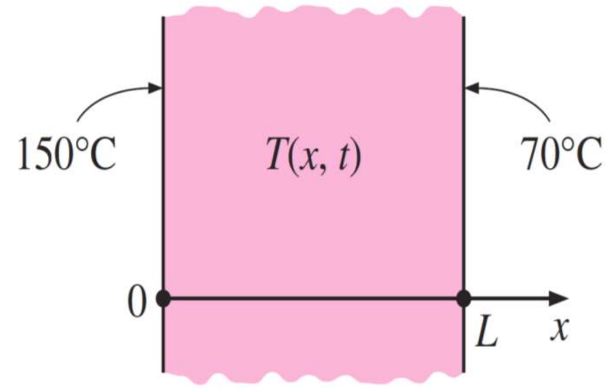
(1) Temperature BC—1st kind
(2) Heat flux BC—2nd kind
(3) Convection BC—3rd kind
(4) Interface BC

4.2.2 Initial and Boundary Conditions

(1) Temperature BC—1st kind

$$T(0, t) = T_1 = 150$$

$$T(L, t) = T_2 = 70$$



Mathematical formulation is given by:

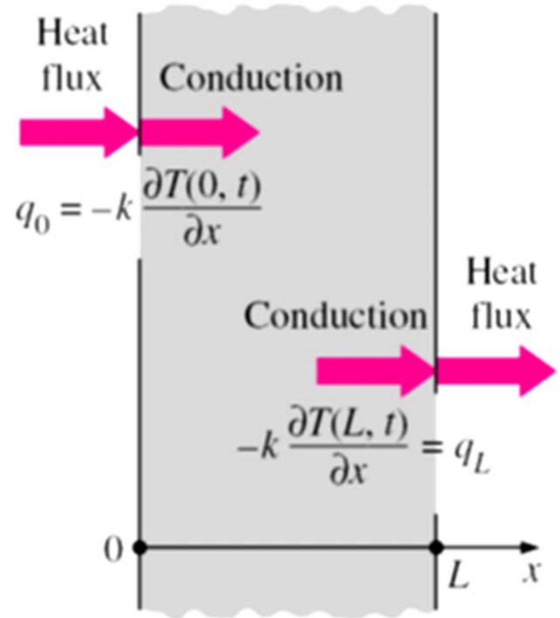
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \ldots \ldots \text{ for } 0<x<L,\ t>0$$

$T = 150$ at $x=0$

$T = 70$ at $x=L$

$T = F(x)$ at $t=0$, $0 \le x \le L$

4.2.2 Initial and Boundary Conditions



(2) Heat Flux BC—2nd kind

Values of q_0 and q_L are known.

$$-k\frac{\partial T(0,t)}{\partial x} = q_0$$

$$-k\frac{\partial T(L,t)}{\partial x} = q_L$$

Mathematical formulation is given by:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$ …… *for 0<x<L, t>0*

$-k\frac{\partial T}{\partial x} = q_0$ *at x=0*

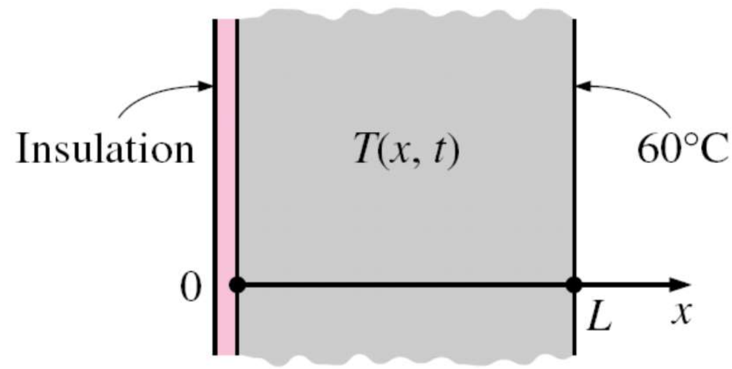
$-k\frac{\partial T}{\partial x} = q_L$ *at x=L*

$T = F(x)$ *at t =0, 0≤x ≤ L*

4.2.2 Initial and Boundary Conditions

(2) Heat Flux BC—2nd kind

Insulated boundary



Mathematical formulation is given by:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$ … … *for 0<x<L, t>0*

$\frac{\partial T}{\partial x} = 0$ *at x=0*

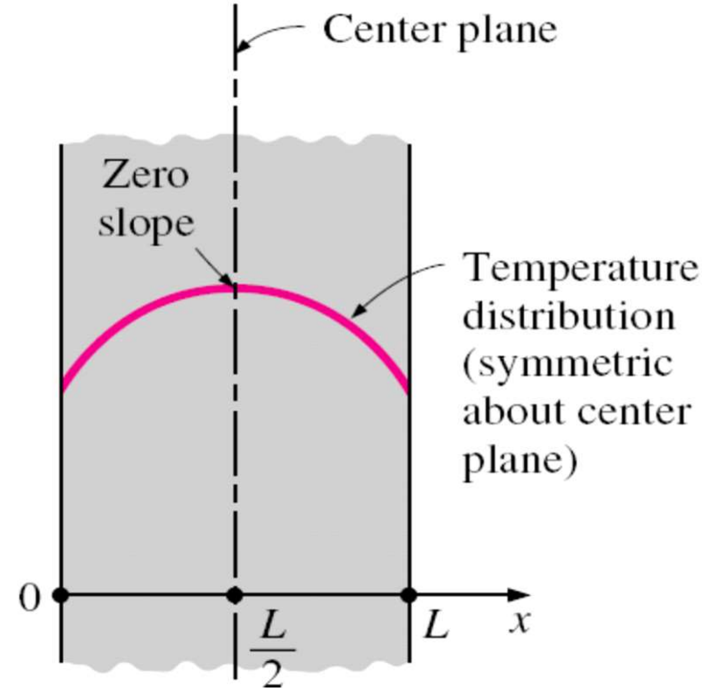
$\mathrm{T} = 60$ *at x=L*

$T = F(x)$ *at t =0, 0≤x ≤ L*

4.2.2 Initial and Boundary Conditions

(2) Heat Flux BC—2nd kind

Thermal symmetry



Mathematical formulation is given by:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$ … … *for 0<x<L, t>0*

$$\frac{\partial T}{\partial x} = 0$$ *at x=L/2*

…..? *at x=L*

….. ? *at t =0, 0≤x ≤ L*

(3) Convection BC—3rd kind

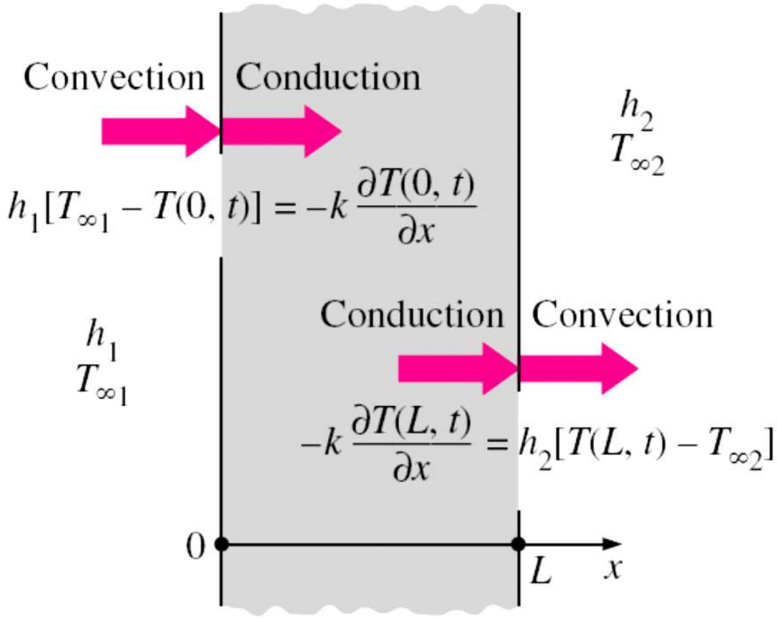
Mathematical formulation is given by:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{…… for } 0<x<L,\ t>0$$

$$-k\frac{\partial T}{\partial x} + h_1 T = h_1 T_{\propto 1} \quad \text{at } x=0$$

$$k\frac{\partial T}{\partial x} + h_2 T = h_2 T_{\propto 2} \quad \text{at } x=L$$

….. ? *at t =0, 0≤x ≤ L*



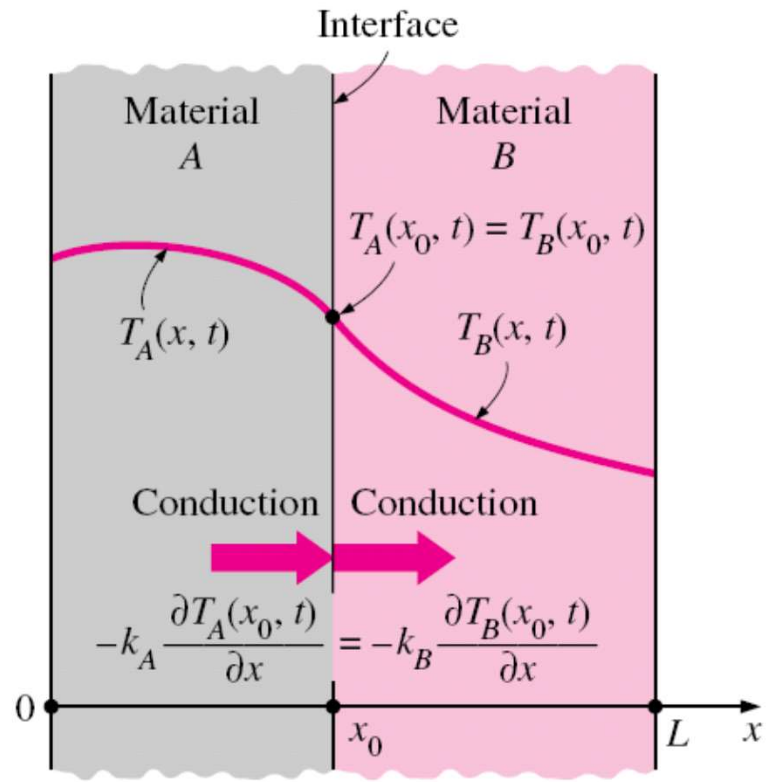
4.2.2 Initial and Boundary Conditions

(4) Interface BC

$$T_A(x_0, t) = T_B(x_0, t)$$

…. ….For perfect contact

$$-k_A \frac{\partial T_A(x_0,t)}{\partial x} = -k_B \frac{\partial T_B(x_0,t)}{\partial x}$$



If the perfect contact is not maintained,

- Contact conductance h_c is important
- $q = h_c(T_A - T_B)$ at x_0